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Interaction of Dislocations with Electrons and with Phonons*

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The interaction parameter of moving dislocations with electrons and with phonons was determined in aluminum, in the temperature range 10 to 250°K. A new technique, involving a "dynamic bias stress", was developed for measuring ultrasonic-attenuation changes $\Delta\alpha$. The numerical values of the interaction parameter were obtained from the measured $\Delta\alpha$ by means of an analysis which does not require any knowledge of the dislocation density or of other inaccurately known features of the dislocation network. The results indicate that the dislocation interaction with electrons is temperature independent and the interaction with phonons increases with increasing temperature. These results are consistent with theoretical predictions.

I. INTRODUCTION

When a dislocation in a crystal is set in motion, it experiences a resistive force against its motion. In the string model of a vibrating dislocation, this frictional force, or damping, is usually assumed to be proportional to the dislocation velocity and is described formally by the following equation:

$$Bv = b\tau, \tag{1}$$

where $b\tau$ is the force per unit length acting on a dislocation moving with velocity v, b is Burgers's vector, τ is the resolved shear stress on the dislocation slip system, and B is the damping constant. This damping constant is assumed here to consist of two parts, one due to interactions of dislocations with phonons ($B_{\rm ph}$) and the other due to interactions with conduction electrons (B_e). It is further assumed that these parts are additive, i.e., $B = B_{\rm ph} + B_e$.

Theoretical attempts to predict the magnitude and temperature dependence of $B_{\rm ph}$ have been made in the past by a number of people including Eshelby,[1] Leibfried,[2] Nabarro,[3] Lothe,[4] Weiner,[5] Mason,[6] and Seeger[7]; and for the electron contribution B_e, by Mason,[8] Kravchenko,[9] Holstein,[10] and Brailsford.[11] Comparison and discussion of some of these predictions are presented by Nabarro.[12]

Commonly used techniques for experimental determination of the damping constant B are (i) direct observation by etch pits[13] or x rays[14] of the displacement of individual dislocations produced by stress pulses of known magnitude and duration, (ii) ultrasonic-attenuation methods,[15–18] and (iii) macroscopic mechanical tests.[19–21] Discussions concerning the relation between the values of damping constants deduced by these methods were given by Fanti *et al.*[22] and by Gillis *et al.*[23] The purpose of this paper is to discuss a new method of obtaining the damping constant by means of ultrasonic measurements and to present experimental values deduced from this approach. With this new method we have overcome many difficulties previously encountered both in the analysis and in the experimental technique. The analysis is applied to the case of aluminum and the value of B thus derived and its temperature dependence are presented.

In Sec. II, the method of analysis is described. The new experimental technique, a dynamic bias stress method, is outlined in Sec. III. The results obtained in aluminum single crystals are given in Sec. IV and

are compared with the existing theories as well as with the values experimentally obtained by other investigators.

II. METHOD OF ANALYSIS

Due to the resistive force [Eq. (1)], dislocations dissipate energy when forced to oscillate by an ultrasonic wave. The corresponding ultrasonic attenuation α_D was derived by Granato and Lücke,[24] using a string analogy of vibrating dislocations

$$\alpha_D = \int_0^\infty \alpha(l) l N(l)\, dl, \tag{2}$$

where $\alpha(l)$, the attenuation per unit length of dislocation, is given by

$$\alpha(l) = (4RGb^2/\pi^2 Av)\{\omega^2 d/[(\omega_0^2-\omega^2)^2+(\omega d)^2]\}, \tag{3}$$

and $N(l)dl$ is the number of dislocations whose loop length lies between l and $l+dl$; ω_0 is the resonant frequency of a dislocation of length l and is given by

$$\omega_0 = (\pi/l)(C/A)^{1/2}; \tag{4}$$

ω is the frequency of the applied ultrasonic wave; $d=B/A$, where B is the damping constant defined in Eq. (1) and A is the effective mass per unit length of dislocation, assumed to be $\pi\rho b^2$, where ρ is the density of the material and b is the Burgers vector; R is an orientation factor; G is the shear modulus; and v is the velocity of the wave. For the distribution function $N(l)dl$, an exponential distribution has been used:

$$N(l)dl = (\Lambda/L_c^2)\exp(-l/L_c)dl, \tag{5}$$

where Λ is the total length of dislocations per unit volume and L_c is the average loop length of the dislocations.

Measurements of α_D, sometimes in conjunction with measurements of the corresponding ultrasonic velocity changes, have been used to determine B.[15–18,22,25] In these approaches it is necessary to determine independently Λ or L_c. For this purpose the dislocation density Λ is usually estimated from etch pit counts. However, the relation between the etch pit count and Λ is not straightforward,[22] and the uncertainty in the value of B thus obtained is quite large.

In order to overcome these limitations, we devised the following technique. Instead of relying on measurements of the total attenuation as a function of frequency, we measure only small changes of attenuation $\Delta\alpha_D$, at each frequency. These result from the application of a small bias stress and are quite reproducible, unlike the total attenuation. From an analysis of this incremental attenuation it is possible to extract the value of B without knowing the dislocation density. The bias stress should be large enough to unpin dislocations from weak pinning points, but be small enough not to generate new dislocations. Since it is expected that the background attenuation is not affected by this small bias stress, the attenuation change obtainable in this method is solely due to the increase in the average loop length of dislocations. The $\Delta\alpha_D-\omega$ plot thus obtained is matched, using a computer, with the theoretical relation discussed later. It turns out that the shape of the $\Delta\alpha_D-\omega$ relation is determined solely by B and ω_0, and it is not necessary to know the value of dislocation density. By repeating the whole sequence at different temperatures, we obtain the temperature dependence of B as well as of ω_0.

The derivation of the frequency dependence of $\Delta\alpha_D$ is as follows: First, it is assumed that dislocations are pinned by two kinds of pins, weak pins and strong pins. The weak pins are capable of holding the dislocation only if the force exerted on them is less than some critical value. If the force exceeds this critical value, the dislocations break away from the pin. The strong pins, on the other hand, fasten the dislocations under all stress values considered here. It is further assumed that both weak pins and strong pins are distributed randomly along the dislocations, and that the condition $L_N/L_c \gg 1$ holds, where L_c and L_N are the average distance between weak pins and between strong pins, respectively.

When a small bias stress is applied to the sample, the dislocations break away from some of the weak pins. If the catastrophic-breakaway model[24] holds, i.e., if one weak pin breaks then all the other weak pins in that segment of loop L_N (between the two strong pins) break catastrophically, one can assume that the breakaway process does not distort appreciably the shape of the distribution of the loop length.[26] Therefore, the effect of the small bias stress on the distribution of loop lengths is a slight increase in the average loop length L_c in expression (5), provided that the condition $L_N/L_c \gg 1$ still holds. Then, the attenuation change $\Delta\alpha_D$ due to that bias stress can be represented, from Eq. (2), by

$$\Delta\alpha_D = \frac{\partial}{\partial L_c}\int_0^\infty \alpha(l) l N_c(l)\, dl\,(\delta L_c)$$

$$= \frac{\Lambda}{L_c^3}(\delta L_c)\int_0^\infty \alpha(l) l\left(\frac{l}{L_c}-2\right)\exp\left(-\frac{l}{L_c}\right)dl. \tag{6}$$

For the integration in expression (6), however, one has to resort to a computer calculation. The main feature here is that the dislocation density Λ and the increment of the average loop length δL_c are outside of the integral, so that the shape of the $\Delta\alpha_D(\omega)$ relation is completely determined by the two parameters ω_0 and B. The effect of a given bias stress on unpinning of dislocations may differ for different temperatures; i.e., the δL_c varies from temperature to temperature even if one applies a constant bias stress at each temperature. This does not affect, however, the above analysis as long as the condition $L_N/(L_c+\delta L_c) \gg 1$ holds.

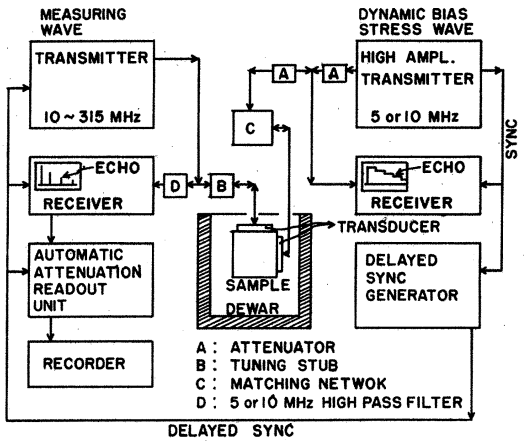

FIG. 1. Block diagram of the electronic equipment; the positions of the "bias wave" and "measuring wave" transducers are also shown.

III. EXPERIMENTAL PROCEDURE

The sample used in this study is an aluminum single crystal 4×4×10 mm in size and is approximately 99.99% in purity. The sample is prepared from a single crystal grown from the melt, and is oriented so that ⟨100⟩ directions are normal to all the faces.

Two waves are propagated in this sample, at right angles, from transducers mounted (by means of "Nonaq") in the manner shown in Fig. 1. One of these waves is propagated at constant frequency (5 or 10 MHz) and at high amplitude. This wave acts as a "dynamic bias stress".[26a] The other wave is propagated at different frequencies, which are the odd harmonics of the transducer's fundamental frequency (10 or 15 MHz). This wave is used for measuring $\Delta\alpha_D(\omega)$ and is referred to as the "measuring wave."

Two echoes are selected out of the echo train of the measuring wave, and the attenuation change caused by the dynamic bias stress is measured, at each frequency, by the change in the amplitude of the second echo relative to the first echo, the amplitude of which is kept constant by means of an automatic gain control. The attenuation change thus obtained is recorded automatically using a recording unit described elsewhere.[27]

As a bias stress mentioned in Sec. II, one can use, in principle, a static bias stress such as a dead weight or a force through a calibrated spring, either in tension or compression. In practice, however, it was found that the reproducibility of $\Delta\alpha_D$ obtained by using a static bias stress was not satisfactory. This may probably be attributed to small amounts of creep, especially at the higher temperatures, as well as to mechanical misalignments. Therefore, all the data reported in this study were taken under the dynamic bias stress with which reproducibility is significantly improved. The dynamic bias pulse is synchronized with the measuring wave pulse. In addition a delay circuit is inserted in the synchronizing circuit, so that one can delay the measuring wave relative to the dynamic bias wave to prevent the radiated signal of the large amplitude bias wave from interfering in the receiver with the early echoes of the measuring wave. The bias pulse amplitude must be large enough to cause unpinning of dislocations from weak pinning points and its width (duration of the pulse) must be long enough to cover the time period a measuring pulse takes to make at least one round trip in the sample.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

A. Effects of Bias Wave

The measuring wave and dynamic bias stress wave do not interact unless the sample has nonlinear characteristics in the stress-strain relation. It was shown previously[28] that two important sources of nonlinearity in a crystal are lattice anharmonicity and nonlinear motion of dislocations. The order of magnitude of the nonlinearities from the two sources is the same, as was determined from their contribution to the harmonic generation of ultrasonic waves. The effect we have observed in the present study, however, cannot be attributed to direct interaction of the two waves through those nonlinearities. This statement is based on the fact that an aluminum sample of the same shape, but containing an appreciable concentration of impurities (2*S* aluminum), did not show any measurable attenuation change upon application of the dynamic bias stresses. Therefore, we conclude that the effect we have observed in this investigation with the high-purity aluminum single crystal is due to the unpinning of dislocations (first-order effect).

The analysis used in this study is valid only if the measuring wave does not cause breakaway of dis-

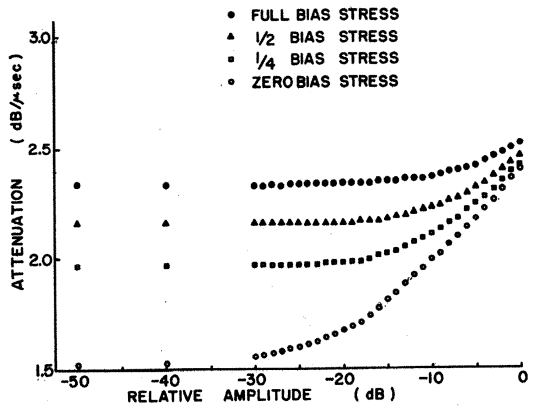

FIG. 2. Amplitude dependence of the attenuation at three levels of bias stress and without bias stress.

locations from pinning points, i.e., there is no hysteretic-loss component in the attenuation.[24]

In examining this problem it should be noted that: (i) The amplitude of the measuring wave is much smaller than that of the bias stress wave; (ii) the frequencies of the measuring waves are much higher than that of the bias stress wave; and (iii) the phase relationship between the measuring and bias stress waves is random. It can be assumed, therefore, that even in the presence of the dynamic bias stress, the contribution from the hysteretic loss to the attenuation of the measuring wave is negligible. In order to verify this assumption, the following experiments are performed. First, the attenuation of the measuring wave without the bias stress is measured as a function of its amplitude at a given frequency. This determines the amplitude level at which the amplitude dependence of the attenuation due to the breakaway process starts to take place. Next, the same procedure is repeated but under the influence of a given bias stress. Typical results (45 MHz for measuring wave, 10 MHz for bias stress wave, at 100°K) are shown in Fig. 2. As can be seen, the amplitude dependence of the measuring wave sets in at an amplitude of approximately −30 dB (−30 dB means that the amplitude of the wave is reduced by 30 dB from the maximum amplitude attainable with the present apparatus) for the case of zero bias stress, while the amplitude-independent region extends to much higher amplitudes in the presence of the bias stresses. Therefore, as long as the amplitude of the measuring wave is kept below the level at which the amplitude dependence sets in without bias stress, the present method is valid.

B. Attenuation Changes: $\Delta\alpha$

The qualitative characteristics of expression (6) are shown in Fig. 3, where $\Delta\alpha_D(\nu)$ is plotted (in arbitrary units) as a function of frequency ν, with B as a parameter. As can be seen, each $\Delta\alpha_D(\nu)$ curve has a maximum, and the position of the maximum moves toward higher values of ν as B decreases. This means

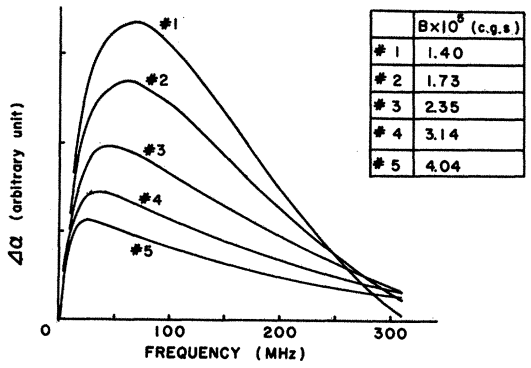

FIG. 3. Incremental attenuation $\Delta\alpha_D$ as obtained from a "model" calculation, using the values of B indicated and $\nu_0=300$ MHz.

that, since B is expected to decrease with decreasing temperature, the maximum in $\Delta\alpha_D(\nu)$ should occur at higher frequencies as temperature decreases. This characteristic is opposite to what one expects in the relaxation type maximum. In this "model" computation, the resonant frequency $\nu_0(=\omega_0/2\pi)$ is assumed to be temperature independent and be equal to 300 MHz with $A=\pi\rho b^2=6.94\times10^{-15}$ g/cm. In actual measurements, ν_0 was found also to be temperature dependent, and consequently, the shift of the maximum with temperature seemed to be enhanced.

Typical results at (16, 72, 129, and 239°K) are shown in Fig. 4. Here, points correspond to the experimentally obtained values of $\Delta\alpha_D(\nu)$, and the solid curves represent the least-squares fit of expression (6) where B and ν_0 are used as fitting parameters. The frequency range covered depended on temperature. In the temperature interval 50 to 240°K the highest frequency for which reproducible results were obtained was 315 MHz. At lower temperatures the interaction of ultrasonic waves with conduction electrons becomes large. Under the conditions of this experiment (i.e., $q\cdot l<1$, where q is the wave number of the ultrasonic wave and l is the mean free path of electrons) the attenuation due to electrons increases as the square of the frequency. Hence, for $T<70$°K the highest frequency used was gradually reduced and at $T\simeq10$°K the highest frequency was only 90 MHz. For $T>250$°K the reproducibility of results was not satisfactory. The difficulties encountered at these temperatures are attributed to changes in the characteristics of the transducer-sample bond.

Each set of $\Delta\alpha_D(\nu)$ data is analyzed by means of expression (6), and the damping constant B and the corresponding average resonant frequencies ν_0 thus obtained are plotted as a function of temperature in Figs. 5 and 6, respectively.

C. Damping Constant B

As can be seen in Fig. 5, B is essentially temperature independent from the lowest temperature explored to approximately 50°K, and increases with increasing temperature thereafter. As mentioned earlier, two contributions to the damping constant are considered here, i.e., $B_{\rm ph}$ and B_e. The theoretical treatments of Holstein,[10] by Kravchenko[9] and by Brailsford,[11] predicted that the electronic damping B_e is temperature independent. This prediction is supported experimentally by our previous work[29] where, in lead, in the normal state, the electronic damping B_e was shown to be temperature independent in the range from 4.2 to 15°K. The damping due to phonons $B_{\rm ph}$, on the other hand, is expected to increase with temperature, starting from values which are much less than B_e at low temperatures. Therefore, the temperature-independent value of B observed below 50°K in this study can be attributed to the electronic damping B_e, and the difference between the temperature-

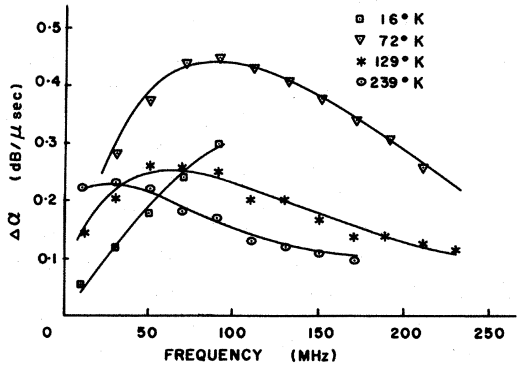

FIG. 4. Incremental attenuation $\Delta\alpha_D$ as a function of frequency for four representative temperatures. The points represent experimental data; the continuous curves represent computer fits of Eq. (8) to these points.

dependent values of B and the B_e just mentioned can be regarded as the phonon damping $B_{\rm ph}$.

For the phonon damping $B_{\rm ph}$, we consider two mechanisms: (i) dissipation of phonon energy due to the fluttering of the dislocation core and (ii) the scattering of phonons by the strain field of a dislocation.

The fluttering mechanism was first studied by Leibfried[2] for the case of an isotropic solid, with the following result:

$$B_{{\rm ph},L}=\sigma\bar{\epsilon}/10c_t,$$

where $\bar{\epsilon}$ is the thermal phonon energy density, c_t is the shear wave velocity, and σ is the scattering width of the dislocation. If the Debye approximation is made for the energy density, this expression becomes

$$B_{{\rm ph},L}=K_1\int_0^{\omega_D}\omega_{\rm ph}^3\,d\omega_{\rm ph}\left[\exp\left(\frac{\hbar\omega_{\rm ph}}{kT}\right)-1\right]^{-1}, \qquad (7)$$

with

$$K_1=\tfrac{1}{10}(\sigma/c_t)3\hbar/2\pi^2c^3,$$

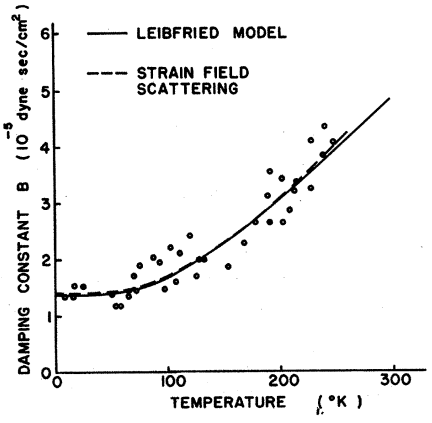

FIG. 5. Damping parameter B as a function of temperature. Open circles represent the values of B obtained from the curves of $\Delta\alpha_D(\omega)$. The solid and dashed curves represent computer fits of Eqs. (9) and (10), respectively, to these points.

where $\omega_{\rm ph}$ is the phonon frequency, c is an appropriate average velocity, T is the absolute temperature, and $\hbar$ and k have the usual meaning. On deriving the above expression, Leibfried assumed, among other things, that (i) a dislocation is divided into segments of atomic length delineated by the lattice planes normal to the dislocation line and (ii) the scattering width is independent of phonon frequency $\omega_{\rm ph}$. From these assumptions, the retarding force on a dislocation moving with velocity v is derived. This force arises from the asymmetry in the phonon distribution due to a Doppler shift in the direction of dislocation motion. This retarding force is related to the damping constant $B_{\rm ph}$ through Eq. (1). Because of the assumption (i), Leibfried's treatment is more applicable to a dislocation kink than to a dislocation line. Considering

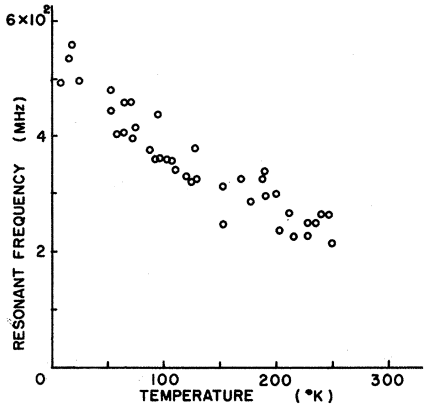

FIG. 6. Resonant frequency ν_0 as a function of temperature; points obtained from the curves of $\Delta\alpha_D(\omega)$.

the motion of a kink, Eshelby[1] made a detailed study of the retarding force caused by the dissipation of phonon energy. Eshelby's treatment yields the result that the scattering width is independent of phonon frequency at low frequencies [supporting assumption (ii) in Leibfried's treatment] and depends on the kink width. Lothe[4] also considered this problem and stated that only the thermal waves of wavelength $\lambda>2w$ (where w is the kink width) should be considered because only those waves make the entire kink oscillate as a whole and radiate energy. More recently, Brailsford[30] extended Eshelby's calculation. In doing so, he took into account the idea of excluding short-wavelength phonons as suggested by Lothe. He showed also that in the limit of "zero" kink width his result reduces to that of Leibfried.

The scattering width responsible for the scattering of phonons by the strain field around a stationary dislocation has been calculated by many investigators including Klemens,[31] Ziman,[32] Carruthers,[33] Lothe,[4] and Ohashi.[34] The results all agree in that the scattering width is proportional to phonon frequency $\omega_{\rm ph}$. From

these scattering widths, $\sigma(\omega_{\rm ph})=\Gamma\omega_{\rm ph}$, one can calculate the retarding force and therefore the damping constant $B_{{\rm ph},S}$ in a manner similar to that used by Leibfried (Doppler shift). The result is

$$B_{{\rm ph},S}=K_2\int_0^{\omega_D}\omega_{\rm ph}{}^4\,d\omega_{\rm ph}[\exp(\hbar\omega_{\rm ph}/kT)-1]^{-1} \qquad (8)$$

with

$$K_2=\Gamma 3\hbar/2\pi^2c^3,$$

where K_2 and Γ are temperature-independent constants, the values of which depend on the method of calculation.

In both cases (i.e., $B_{{\rm ph},L}$ and $B_{{\rm ph},S}$) $B_{\rm ph}$ increases linearly with temperature at high temperatures and changes more rapidly at low temperatures. According to Brailsford's treatment[30] the onset of the high-temperature region depends on the cutoff frequency, i.e., on the width of the kink.

With this theoretical background, the experimental results (Fig. 5) are analyzed in the following way. A least-squares fit is obtained for

$$B=B_e+B_{\rm ph}$$

to the experimental data. In the fitting procedure, B_e and either K_1 or K_2 in $B_{\rm ph}$ are used as adjustable parameters. It should be noted that B_e, K_1, and K_2 are temperature independent. The temperature dependence of $B_{\rm ph}$ is contained in the integrals of expressions (7) and (8). In Fig. 5 the solid and dashed curves show the fit using, respectively, expressions (7) and (8). At the point of maximum deviation the difference between the two curves is only $\sim 2\%$. It is obvious that the scatter of the experimental points is much too large to permit any meaningful distinction between the two theoretical approaches. It is possible, however, to calculate the scattering width σ of the dislocations from parameters associated with the curves of Fig. 5. In the case of expression (7), the value of K_1 obtained from the fit is 8.05×10^{-59} cgs units and the corresponding $\sigma=3.5\times10^{-8}$ cm, if it is assumed that $c_t=c=3\times10^5$ cm/sec. In the case of expression (8), $K_2=7.23\times10^{-67}$ cgs units and $\Gamma=2.72\times10^{-20}$ cm sec [in $\sigma(\omega)=\Gamma\omega_{\rm ph}$] are obtained, again by assuming $c=3\times10^5$ cm/sec. Ohashi's calculation[34] of the dislocation scattering width yields

$$\Gamma_{\rm Ohashi}=\beta_1(-g/\rho c^2)^2b^2/c,$$

where g is a combination of second- and third-order elastic constants, and β_1 is a numerical constant which depends on the orientation of the dislocation relative to the incident phonons and on the dislocation characteristics (i.e., edge, screw, or mixture). The calculations of Klemens and Ziman yield

$$\Gamma_{K;Z}=\beta_2\gamma^2\,b^2/c,$$

where γ is a Grüneisen constant and β_2 is similar in character to β_1. The value of Γ quoted above corresponds to $g(\beta_1)^{1/2}=1.2\times10^{12}$(cgs) and $\gamma(\beta_2)^{1/2}=3$. The constants β_1 and β_2 are expected to deviate from unity by not much more than an order of magnitude, hence the values of g and γ are physically reasonable. Thus both Leibfried's theory and the strain field method yield values of σ which are physically plausible [note that for $\omega_{\rm ph}=10^{12}$ cps, $\sigma(\omega)=2.72\times10^{-8}$ cm] and consistent with Leibfried's estimate that σ should be of the order of one Burgers vector.

The electronic damping B_e has been treated by several authors within the framework of free-electron theory. Mason[8] assumed that the damping was due to the "viscosity" of the conduction-electron gas and derived an expression for B_e of the form

$$B_e=P/\rho,$$

where P is a product of fundamental and material constants, and ρ is the electrical resistivity of the material. Huffman and Louat[35] obtained a similar form of B_e (B_e inversely proportional to ρ) from a solution of the Boltzmann transport equation. Both treatments yield a temperature-dependent B_e through the temperature dependence of ρ. In contrast to these treatments the calculations of Holstein[10] and of Kravchenko,[9] as mentioned before, resulted in temperature-independent expressions for B_e. Holstein treated the interaction of the Fourier components of a moving dislocation strain field with conduction electrons by first-order perturbation theory. Kravchenko treated the same interaction by means of the Boltzmann transport equation. Both authors obtain essentially identical expressions for B_e. More recently Brailsford[11] refined these calculations and also obtained a temperature-independent expression for B_e. He also pointed out that the temperature dependence of B_e obtained by Huffman and Louat was due to an error in their treatment. In the case of edge dislocations Brailsford obtains the following expression for B_e:

$$B_e=\left(\frac{1-2\nu}{1-\nu}\right)\frac{n_0mv_Fb^2q_D}{96}\,\phi\left(\frac{q_D}{q_{\rm TF}}\right), \qquad (9)$$

where

$$\phi(x)=\tfrac{1}{2}[(1+x^2)^{-1}+x^{-1}\tan^{-1}x].$$

In the above, ν is Poisson's ratio, n_0 is the electron density in equilibrium, m is the free-electron mass, v_F is the Fermi velocity, q_D is the radius of the Debye sphere, and $q_{\rm TF}$ is the reciprocal of the Thomas-Fermi screening length. Except for some numerical factors, this expression is identical with those of Holstein and Kravchenko. As mentioned previously, the experimental results shown in Fig. 5 are fitted to $B=B_e+B_{\rm ph}$, where B_e is an adjustable parameter determined simultaneously with K_1 or K_2. The best fit is obtained with $B_e=1.40\times10^{-5}$ dyne sec/cm^2, whereas the value obtained for edge dislocations in aluminum, using expression (9) is 1.5×10^{-5} dyne sec/cm^2. The agreement between the theoretical and experimental values of B_e is quite close. It is suggested here, therefore, that the theoretical treatments which yield

this value of B_e are essentially correct even though they are based on free-electron theory and include several other approximations. Furthermore, the present experimental results on aluminum and previously reported results on lead[29] are clearly consistent with a temperature-independent B_e. It is concluded, therefore, that the results of Mason's and of Huffman and Louat's calculations are not supported by these experiments.

In order to compare the method of analyzing data used in this study with previously used approaches (e.g., deriving B values from the high-frequency limit of decrement combined with the etch pit count of dislocation density), we analyzed, using our method, the data on Cu, and a Cu-0.13% Mn alloy obtained by Suzuki, Ikushima, and Aoki.[16] Since the data in their paper were presented as decrement $\Delta_D=(2\pi/\omega)\alpha_D$ and not the incremental attenuation $\Delta\alpha_D$, Eq. (2) is used for the computer fitting. Two sets of data on copper for each of two different single crystals, Cu No. 1 and Cu No. 2, are given by Suzuki *et al.*[16] The room-temperature values of B quoted by these authors for these two copper samples are 2.7×10^{-5}(cgs) and 1.2×10^{-4}(cgs), respectively; i.e., they differ by about a factor of 4.5. It is difficult to understand why the values of B should be vastly different for two different samples of the same material, copper, at the same temperature. Our results on the other hand are 1.1×10^{-4}(cgs) and 1.35×10^{-4}(cgs), respectively, which are the same within the uncertainty of the method. The temperature dependence of B for Cu No. 2 agrees qualitatively with our analysis (there are no data reported on the temperature dependence of B for Cu No. 1.) For the Cu-0.13% Mn alloy, the values of B obtained by our analysis are considerably smaller than their values and are close to the values of the pure copper crystals. If the addition of Mn to Cu does not affect the stacking fault energy of dislocations as suggested by these authors, the value of B should not change much from that of copper. The resonant frequencies, on the other hand, are definitely larger in the alloy than in the copper crystals, according to our analysis. This indicates a shortening of dislocation loop length due to pinning by the added impurity, as expected.

The damping constant has been measured in aluminum by several authors, including Gorman, Wood, and Vreeland,[14] Mason and Rosenburg,[36] and Ferguson, Kumar, and Dorn.[21] Mason and Rosenburg[36] (from ultrasonic-attenuation measurements) and Ferguson, Kumar, and Dorn[21] (from stress-strain rate measurements) reported values of B and their temperature dependence for aluminum. In both cases, however, no attempts were made to establish the dislocation density Λ of the samples, on which the determination of the value of B critically depends. Therefore, the absolute values of B derived in those studies cannot be compared with the results of this study. The temperature dependence of B reported in Refs. 36 and 21 conflicts with our results.

As far as the authors are aware, the only experimental results reported on B for aluminum, in which all sample parameters were measured, are those of Gorman, Wood and Vreeland.[14] These investigators used the direct measurement of dislocation displacements carried out by an x-ray technique. The values of B quoted in their work are considerably larger than the results obtained in this study, i.e., approximately $B=1.45\times10^{-4}$(cgs) at 123°K and 2×10^{-4}(cgs) at 170°K. In their technique the damping constants are deduced from the relation

$$B=\tau b/v,$$

where v is the velocity of dislocations. Since the velocity v is determined from d/t, d being the distance the dislocation has moved during a stress pulse of duration t, it does not necessarily represent the instantaneous velocity of dislocations controlled only by the damping. The results may include the effects of inertia and line tension of dislocations as well as the effects of pinning points on and along the path of dislocations. It is possible, therefore, that the velocity thus determined could be an underestimate for the instantaneous velocity, which in turn results in an overestimate of the damping constant B discussed in this study. Although only three values of B are presented for the temperature range from 123 to 373°K in their work, the temperature dependence of B appears to agree with the results of the present study.

D. Average Resonant Frequency

As seen in Fig. 6, the average resonant frequency prior to breakaway due to the bias stress varies with temperature. Since C (line tension) and A (effective mass of dislocations) are expected to be insensitive to temperature, the data in Fig. 6 indicate that the loop length L_c becomes shorter as temperature decreases. This result can be explained qualitatively by considering the thermal breakaway of dislocations. As temperature increases, dislocations may break away, with the help of thermal energy, from some of the pinning points even under the small stress of the measuring waves. It is likely, therefore, that the loop length L_c seen by the measuring wave becomes larger as the temperature increases. However, a quantitative analysis of $\nu_0(T)$ is not possible at present.

Similar temperature dependence of ν_0 is obtained with the present analysis for the data of copper and Cu-0.13% Mn alloy reported by Suzuki *et al.*[16]

V. CONCLUSIONS

(i) The interaction parameter B of moving dislocations with electrons and with phonons in aluminum has been determined in the temperature range 10 to 250°K.

(ii) A new technique of measuring changes of

ultrasonic attenuation $\Delta\alpha$, as a function of a dynamic bias stress, was developed. This technique eliminates the need to know the absolute value of the attenuation due to the particular mechanism under study.

(iii) The values of B were obtained from an analysis of $\Delta\alpha$, measured as a function of dynamic bias stress and of frequency. This method does not depend on a knowledge of the dislocation density or of other unknown, or difficult to determine, features of the dislocation network.

(iv) The results indicate that B_e, the part of B due to interactions with electrons, is independent of temperature, as discussed by Holstein,[10] Kravchenko,[9] Elbaum and Hikata,[37] and Brailsford.[11] The numerical value of B_e is in close agreement with the value obtained from the calculations of Brailsford.

(v) $B_{\rm ph}$, the part of B due to interactions with phonons, increases with increasing temperature in a manner consistent with several theories. These theories are based on different mechanisms, but lead to very small differences in the temperature dependence of $B_{\rm ph}$ and cannot be distinguished on the basis of the present results. However, the phonon scattering width (cross section per unit length) of dislocations deduced from the present measurements are quite close to those expected from physical and theoretical arguments.

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